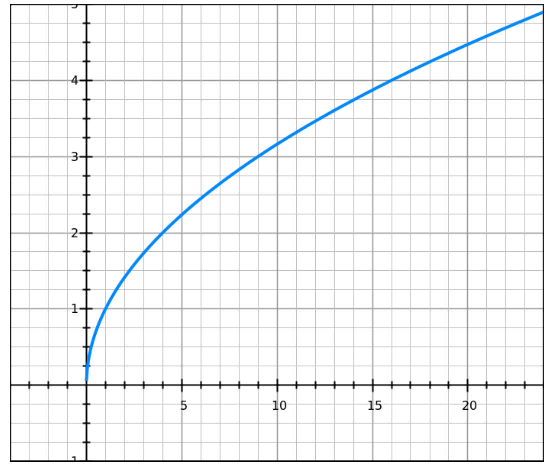
The definite integral Class 33



1. The figure shows a plot of $y = \sqrt{x}$.

   (a) Use the figure to estimate $\int_0^{15} \sqrt{x}\, dx$.

   (b) For this integral, are left sums always overestimates, always underestimates, or could they be either? What about right sums?

   (c) Use the spreadsheet to estimate $\int_0^{15} \sqrt{x}\, dx$ to 2 significant figures. Use the formula `sqrt(x)`.

   (d) Explain how you know your answer to (c) is correct to the required accuracy. Explain why you had to use $n \geqslant 131$ to be sure of your result.

   (e) In fact, $\int_0^{15} \sqrt{x}\, dx = 38.72983$, to 7 significant figures. Using a central sum with $n \geqslant 131$, get another estimate for the integral. How accurate is the new estimate? Why is it more accurate?

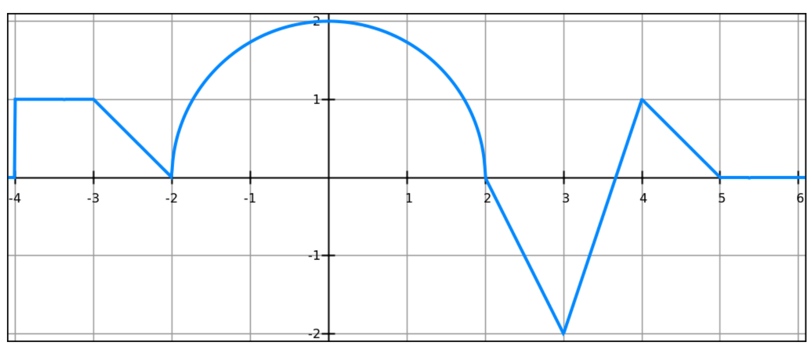
2. Use your calculator to plot $y = e^{-x}\sin(x)$. Is $\int_0^{2\pi} e^{-x}\sin(x)\, dx$ positive or negative?

3. (a) Sketch $y = \sin(x)$ and use your sketch to find $\int_0^{2\pi} \sin(x)\, dx$.

   (b) Use the spreadsheet to estimate $\int_0^{\pi} \sin(x)\, dx$. Use the formula `sin(x)`, and enter `=PI()` for xmax.

   (c) In $\int_0^{\pi} \sin(x)\, dx$, are left sums always overestimates, always underestimates, or does it depend on n? What about right sums? Center sums?

   (d) Hence estimate $\int_0^{5\pi} |\sin(x)|\, dx$.



4. The plot shows $y = g(x)$. Evaluate

   (a) $\int_{-4}^{5} g(x)\, dx$

   (b) $\int_{-4}^{5} |g(x)|\, dx$

5. Let $g(t) = \int_0^t \frac{x^2}{3}\, dx$.

   (a) Enter the formula `x^2/3` into the spreadsheet and use it to estimate $g(t)$ for $t = 1, 2, 3, 4$.

   (b) Suggest a simple formula for $g(t)$.

   (c) Test your formula from (b) by using the spreadsheet to estimate $g(0.5)$, $g(2.5)$.