Distance travelled

1. The velocity $v(t)$ in the table below is increasing for $0 \leqslant t \leqslant 12$.

t	0	3	6	9	12
$v(t)$	34	37	38	40	45

   (a) Find an upper estimate for the total distance traveled using $n = 2$.

   (b) If you were to find an upper estimate for the total distance travelled using $n = 4$, would this be more or less accurate than your estimate in (a)?

   (c) Find a lower estimate of the total distance travelled using $n = 4$.

2. In this problem, all distances, speeds, etc. are relative to the ground. The Cincinnati steampunk band *Automaton* are planning to use an ornithopter to travel between gigs on their tour of the Midwest. After reaching a safe altitude, the ornithopter accelerates uniformly from 0mph to 40mph, travelling 4 miles as it does.

   (a) Sketch the horizontal velocity of *Automaton's* ornithopter as a function of time.

   (b) Hence calculuate the time it took to reach 40mph.

   The ornithopter is driven by peddle power, so its speed reduces when a roadie gets tired, and increases when that roadie is replaced by another. The ornithoptor's speed at time t after it reaches 40mph is given by

   $$v(t) = 40 - 8\sin(\pi t).$$

   (c) Sketch $v(t)$.

   (d) Explain why the ornithopter will travel 160 miles in the 4 hours after it first reaches 40mph.

   The band's Chief Tinkerer (also drummer and backing vocalist) is working on a way to drive the ornithopter more efficiently, so that instead it will have speed

   $$V_k(t) = 40 - 6\sin(\pi t) + kt$$

   at time t hours after it first reaches 40mph, where $k > 0$.

   (e) Sketch $V_2(t)$.

   (f) How far will *Automaton's* new and improved ornithopter travel in the 4 hours after it first reaches 40mph, assuming $k = 2$?

   (g) The band need to fly 200 miles from Chicago to Ann Arbor in the 4 hours after the ornithopter first reaches 40mph. What value of k must the Chief Tinkerer achieve?

3. Use the expressions for left and right sums on page 276 of the textbook, and the following table:

t	0	4	8	12	16
$f(t)$	25	23	22	20	17

(a) If $n = 4$, what is Δt? What are t_0, t_1, t_2, t_3, t_4? What are $f(t_0), f(t_1), f(t_2), f(t_3), f(t_4)$?

(b) Find the left and right sums usinf $n = 4$.

(c) If $n = 2$, what is Δt? What are t_0, t_1, t_2? What are $f(t_0), f(t_1), f(t_2)$?

(d) Find the left and right sums using $n = 2$.

The *central sum* is defined by

$$\text{Total distance traveled between } t = a \text{ and } t = b \approx f\left(\frac{t_0 + t_1}{2}\right)\Delta t + f\left(\frac{t_1 + t_2}{2}\right)\Delta t + f\left(\frac{t_2 + t_3}{2}\right)\Delta t + \ldots + f\left(\frac{t_{n-1} + t_n}{2}\right)\Delta t.$$

(e) Find the central sum using $n = 2$.

(f) Find the central sum using $n = 4$, or explain why it is impossible.