

More distance travelled

Class 32

1. A particle's velocity is given by $v(t)$, where

(i) $v(t) = 12 - 2t$ for $0 \leq t \leq 6$.

(ii) $v(t) = \begin{cases} 2 & \text{if } 0 \leq t < 3 \\ -8 + 2t, & \text{if } 3 < t \leq 6. \end{cases}$

(iii) $v(t) = 3t^2$ for $0 \leq t \leq 6$.

(iv) $v(t) = \cos(t) + t/4$ for $0 \leq t \leq 6$.

For each of (i)–(iv),

- (a) Sketch the function. Use your calculator to help for (iv).
- (b) Calculate the displacement of the particle (signed area under the curve) between $t = 0$ and $t = 6$ using geometry, for (i) and (ii).

The displacement of (iii) is 216, and the displacement of (iv) is approximately 4.220585.

- (c) Use the spreadsheet to find estimates for the displacement of the particle, using left, right, and central sums, for $n = 6$. Record which is an overestimate and which is an underestimate, and use the graph to explain why.
- (d) Do the estimates improve when n is increased? Do they stay over/under estimates or do they change?

Useful formulae for cell C1:

- (i) $12-2*x$
- (ii) $2+(1+\text{sign}(x-3))*(x-5)$
- (iii) $3*x^2$
- (iv) $\cos(x)+x/4$

2. Dagny needs to return to New York, which is 630km away, to save her railroad company from the moochers. She has her private carriage linked up to The Express train, and asks the driver to report the train's speed to her every half hour. At 6pm, the brakeman will stop working and the train will have to stop for safety reasons. The velocity $v(t)$ km/h of Dagny's train is an increasing function of time t hours after 1:00pm. The velocity of the train, as reported by the driver, is given in the table below.

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$v(t)$	0	44.7	83.1	112.0	133.2	147.3	156.7	163.1	167.5	170.5	172.7

- (a) Use upper and lower sums to estimate the distance Dagny's train has travelled at 6pm.
 (b) Has Dagny made it to New York?

Dagny's colleague Eddie knows that Dagny's train will be travelling at

$$v(t) = \frac{360}{\pi} \left[\arctan\left(\frac{2t}{5}\right) + \frac{2t}{5 + \frac{4t^2}{5}} \right].$$

Eddie enters this function into the spreadsheet as

$$(\text{atan}(2*x/5)+2*x/(5+4*x^2/5))*360/PI()$$

and calculates how far Dagny's train can travel by 6pm.

- (c) Will Dagny arrive in New York?
 (d) How many steps does Eddie need to use to be sure of his answer?
3. Find the difference between the left sum estimate and the right sum estimate for the displacement at velocity $v(t) = 3t + 4$ on the interval $a \leq t \leq b$ for n subdivisions.